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Fine tuning as an indication of physics beyond the MSSM

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Abstract

We investigate the amount of fine tuning of the electroweak scale in the presence of new physics beyond the MSSM, parametrized by higher dimensional operators. We show that these significantly reduce the MSSM fine tuning to $\Delta < 10$ for a Higgs mass between the LEPII bound and 130 GeV, and a corresponding scale M_* of new physics as high as 30 to 65 times the Higgsino mass. If the fine-tuning criterion is indeed of physical relevance, the findings indicate the presence of new physics in the form of new states of mass of $\mathcal{O}(M_*)$ that generated the effective operators in the first instance. At small $\tan\beta$ these states can be a gauge singlet or a $SU(2)$ triplet. We derive analytical results for the EW scale fine-tuning for the MSSM with higher dimensional operators, including the quantum corrections which are also applicable to the pure MSSM case in the limit the coefficients of the higher dimension operators vanish. A general expression for the fine-tuning is also obtained for an arbitrary two-Higgs doublet potential.

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1 Introduction.

Low-energy supersymmetry offers an elegant solution to the hierarchy problem. One consequence of this is that it introduces a spectrum of supersymmetric states in the visible sector with mass of the order of the electroweak scale. However, none of the superpartners of the Standard Model have been seen, although there is hope that LHC will soon remedy this. In trying to determine the physics beyond the Standard Model the fact that no superpartners have been observed is significant as it (re)introduces the need for some amount of fine tuning of the parameters of the minimal supersymmetric standard model (MSSM), to separate the electroweak and supersymmetry breaking scales (the "little hierarchy problem"). On the other hand circumstantial evidence for supersymmetry such as the successful unification of couplings [1, 2, 3, 4] or radiative electroweak breaking [5] is consistent with and in fact requires the existence of such light superpartners.

The basic issue raised by fine-tuning is the sensitivity of the electroweak scale (more precisely the mass of Z) to small variations of the input parameters of the MSSM, consistent with the measured Z mass, m_Z, and the current bounds on the lightest Higgs mass, m_h. The need to keep fine tuning small indicates a light Higgs in some tension with the LEPII bound [6] $m_h \geq 114.4$ GeV. Consistency of this bound with the MSSM tree level bound $m_h \leq m_Z$ can only be achieved at the quantum level, by a large top quark/squarks loop correction to m_h. To maximise this, the top squarks must be quite massive or highly mixed implying that the MSSM model is fine-tuned.

Various definitions of fine tuning have been proposed. The most popular one [7] is based on the logarithmic derivatives of the observables with respect to the set of parameters considered. It has been widely used in quantifying the fine tuning in the MSSM [8, 9, 10, 11, 12, 13, 14] and we shall use it in this analysis. The new feature of our analysis starts with the premise that if low-energy supersymmetry is indeed the solution to the hierarchy problem, a significant amount of fine tuning of the electroweak scale in the MSSM may in fact suggest that there are additional new degrees of freedom in the theory beyond those of the MSSM. There are many models that consider additional degrees of freedom beyond the MSSM in order to reduce the amount of fine tuning. The NMSSM is just such an example [15] which has an extra chiral singlet. One can consider other MSSM extensions with more chiral fields, additional gauge interactions, etc. Each of these brings different solutions to the little hierarchy problem and it is difficult to assess which of these is the most compelling. In this paper we perform a model

independent analysis of the nature of this new physics based on a general parametrisation of physics beyond the MSSM. In particular we extend the MSSM by the addition of higher dimensional operators [16, 17, 18, 19] that encode the effect of all possible new physics at scales below the appearance of the new degrees of freedom. Having identified the most relevant operators one can later address the question of what new physics generated these operators in the first instance. The advantage of the effective approach is that it provides an organising principle according to which one usually restricts the analysis to operators of a given (leading) order in the scale of new physics M_*, with higher order operators suppressed by higher powers of M_*. The analysis we consider includes dimension $d = 5$ and $d = 6$ operators beyond MSSM [20, 21, 23]. For the case of $d = 5$ operators we determine the amount of fine tuning as a function of the mass of the lightest Higgs corrected by the quantum contributions using both analytical and numerical techniques. One-loop renormalisation group corrections in the Higgs potential are also included.

The expectation that higher dimensional operators can reduce the amount of fine tuning is broadly based on two arguments. Firstly these operators may directly increase the tree level value of m_h [20, 21]. Consequently the tree level upper bound on m_h may be relaxed, and the quantum effects needed to satisfy LEPII bound may be smaller, corresponding to reduced fine tuning. Secondly, the higher dimensional operators may generate additional contributions to the quartic Higgs couplings of the MSSM, again serving to increase the Higgs mass. This effect can be quite significant because, in the MSSM, the quartic coupling, $(g_2^2 + g_1^2)/8$, where g_2 and g_1 are the $SU(2) \times U(1)$ gauge couplings, is anomalously small; indeed its smallness is a major source of the little hierarchy problem. For the case of just the $d = 5$ operators a numerical study shows that these effects can reduce the amount of fine tuning, Δ, of the electroweak (EW) scale relative to the MSSM case, to less than 10 for a Higgs mass in the range $114.4\,\mathrm{GeV} \leq m_h \leq 130$ GeV even for a scale of new physics as high as (30 to 65) times the higgsino mass, and possibly above the LHC reach. We also give in Appendix an analytical formula for the EW fine-tuning in a general two-Higgs doublet model, which can be easily applied to specific models.

The plan of the paper is as follows. Section 2 lists the $d = 5$ and $d = 6$ operators that are consistent with the MSSM symmetries and that can affect fine tuning. In Section 3 we evaluate analytically and numerically the fine tuning in the MSSM extended by the $d = 5$ operators. The conclusions are given in Section 4.

2 Higher dimensional operators beyond MSSM Higgs sector

In this section we list the effective operators of dimension $d = 5, 6$ that can be present in the Higgs sector consistent with the symmetries of the MSSM. These operators parametrise new physics beyond the MSSM and affect the Higgs scalar potential. Therefore they also affect the amount of fine tuning of the EW scale, as discussed in detail in the next section. The ($R-$parity conserving) $d = 5$ operators in the MSSM Higgs sector are:

$$\mathcal{L}_1 = \frac{1}{M_*}\int d^2\theta\, \lambda(S)\, (H_1\, H_2)^2, \qquad (1)$$

$$\mathcal{L}_2 = \frac{1}{M_*}\int d^4\theta\, \Big\{ A(S,S^\dagger) D^\alpha \Big[B(S,S^\dagger)\, H_2\, e^{-V_1}\Big] D_\alpha \Big[C(S,S^\dagger)\, e^{V_1}\, H_1\Big] + h.c.\Big\} \qquad (2)$$

where S is the spurion field, $S = \theta\theta\, m_0$, $A(S,S^\dagger)$, $B(S,S^\dagger)$, $C(S,S^\dagger)$ are polynomials in $S, S^\dagger$ and m_0 is the susy breaking scale in the visible sector (in gravity mediation $m_0 = \langle F_h\rangle/M_{Planck}$ where $\langle F_h\rangle$ is the auxiliary field vacuum expectation value (vev) in the hidden sector responsible for supersymmetry breaking). As we discuss in Section 3.5 the first operator can be generated, for example, by integrating out massive gauge singlets or $SU(2)$ triplets, while the second is easily generated by integrating out a pair of massive Higgs doublets [21], all of mass of order M_*.

In [21, 22] it was shown that by using general field redefinitions one can remove $\mathcal{L}_2$ from the action. The effect of this is an overall renormalisation of the soft terms and of the μ term. Since the fine tuning measure includes the fine tuning with respect to each of these soft operators separately adding $\mathcal{L}_2$ cannot reduce the overall fine tuning. For this reason we will only include $\mathcal{L}_1$ in our discussion of fine tuning with $d = 5$ operators.

There are also $d = 6$ operators that can be present in addition to the MSSM Higgs sector. These are suppressed relative to the $d = 5$ operators by the factor $1/M_*$. However they may give contributions to the Higgs potential enhanced by $\tan\beta$ relative to the $d = 5$ so cannot be ignored at very large $\tan\beta$. The list of $d = 6$ operators is (see also [16, 17]):

$$\mathcal{O}_i = \frac{1}{M_*^2}\int d^4\theta\; \mathcal{Z}_i(S,S^\dagger)\; (H_i^\dagger\, e^{V_i}\, H_i)^2, \qquad i = 1, 2.$$

$$\mathcal{O}_3 = \frac{1}{M_*^2}\int d^4\theta\; \mathcal{Z}_3(S,S^\dagger)\; (H_1^\dagger\, e^{V_1}\, H_1)\, (H_2^\dagger\, e^{V_2}\, H_2), \qquad (3)$$

(These can be generated by integrating a massive $U(1)$ gauge boson or a $SU(2)$ triplet).

$$
\begin{aligned}
\mathcal{O}_4 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_4(S,S^\dagger)\ (H_2\,H_1)\,(H_2\,H_1)^\dagger, \\
\mathcal{O}_5 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_5(S,S^\dagger)\ (H_1^\dagger\,e^{V_1}\,H_1)\,(H_2\,H_1+h.c.) \\
\mathcal{O}_6 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_6(S,S^\dagger)\ (H_2^\dagger\,e^{V_2}\,H_2)\,(H_2\,H_1+h.c.) \\
\mathcal{O}_7 &= \frac{1}{M_*^2}\int d^2\theta\ \mathcal{Z}_7(S,0)\ W^\alpha\,W_\alpha\,(H_2\,H_1)+h.c., \\
\mathcal{O}_8 &= \frac{1}{M_*^2}\int d^4\theta\ \Big[\mathcal{Z}_8(0,S^\dagger)\ (H_2\,H_1)^2+h.c.\Big]
\end{aligned}
\tag{4}
$$

where W_α is the supersymmetric field strength of a vector superfield of the SM gauge group. $\mathcal{O}_4$ can for example be generated by integrating a gauge singlet.

$$
\begin{aligned}
\mathcal{O}_9 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_9(S,S^\dagger)\ H_1^\dagger\,\overline{\nabla}^2\,e^{V_1}\,\nabla^2\,H_1 \\
\mathcal{O}_{10} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{10}(S,S^\dagger)\ H_2^\dagger\,\overline{\nabla}^2\,e^{V_2}\,\nabla^2\,H_2 \\
\mathcal{O}_{11} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{11}(S,S^\dagger)\ H_1^\dagger\,e^{V_1}\,\nabla^\alpha\,W_\alpha\,H_1 \\
\mathcal{O}_{12} &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_{12}(S,S^\dagger)\ H_2^\dagger\,e^{V_2}\,\nabla^\alpha\,W_\alpha\,H_2
\end{aligned}
\tag{5}
$$

where ∇_α acts on everything to the right and $\nabla_\alpha\,H_i = e^{-V_i}\,D_\alpha\,e^{V_i}H_i$. i=1,2. In addition to the spurion dependence in the wavefunctions $\mathcal{Z}_i(S,S^\dagger)$, extra $(S,S^\dagger)$ dependence (not shown) can be present under each derivative ∇_α in eq.(5), in order to ensure the most general supersymmetry breaking contribution associated to these operators. One may use the equations of motion to replace the operators involving extra derivatives by non-derivative ones[1]. Note that when computing the fine tuning measure eliminating a particular operator will lead to correlations between the remaining operators that, strictly, should be taken into account.

Given the large number of $d=6$ operators, determination of the fine tuning with respect to their coefficients is difficult. For this reason we restrict the following discussion to the $d=5$ operators. In Section 3.3 we comment on the new contributions that may come from $d=6$ operators and discuss the limit on our analysis that follows from keeping only $d=5$ operators.

[1] Setting higher derivative operators onshell is a subtle issue in this case. One can also use general spurion-dependent field redefinitions to "gauge away" (some of) these operators, using the method of [21, 22].

3 Fine-tuning in MSSM with d=5 operators (MSSM$_5$).

3.1 The scalar potential

In this section we evaluate the EW scale fine-tuning in the MSSM extended by $\mathcal{L}_1$ in eq.(1). Including it together with the MSSM, the full Higgs Lagrangian is then given by

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \left[(1 - c_{h_1}\, S\, S^\dagger)\, H_1^\dagger\, e^{V_1}\, H_1 + (1 - c_{h_2}\, S\, S^\dagger)\, H_2^\dagger\, e^{V_2}\, H_2 \right] \\ &+ \int d^2\theta \left[\mu_0\, (1 + B_0\, S)\, H_1\, H_2 + \frac{1}{M_*}\, (1 + c_0 S)\, (H_1\, H_2)^2 \right] + h.c. \end{aligned} \tag{6}$$

The corresponding scalar potential is given by

$$\begin{aligned} V &= \tilde{m}_1^2\, |h_1\,|^2 + \tilde{m}_2^2\, |h_2\,|^2 - \big(m_3^2\, h_1\, h_2 + h.c.\big) + \frac{g^2}{8}\, \big(|\, h_1\,|^2 - |\, h_2\,|^2\big)^2 + \frac{g^2}{8}\, \delta\, |\, h_2\,|^4 \\ &+ \big(|\, h_1\,|^2 + |\, h_2\,|^2\big)\, \big(\zeta_1\, h_1\, h_2 + h.c.\big) + \frac{1}{2}\, \big(\, \zeta_2\, (h_1\, h_2)^2 + h.c.\big) \end{aligned} \tag{7}$$

where $g^2 \equiv g_1^2 + g_2^2$,

$$\zeta_1 = 2\, \mu_0^*/M_*, \qquad \zeta_2 = -2\, c_0\, m_0/M_* \tag{8}$$

and

$$\begin{aligned} \tilde{m}_1^2(t) &= m_0^2 + \mu_0^2\, \sigma_8^2(t) + m_{12}^2\, \sigma_1(t) \\ \tilde{m}_2^2(t) &= \mu_0^2\, \sigma_8^2(t) + m_{12}^2\, \sigma_4(t) + A_t\, m_0\, m_{12}\, \sigma_5(t) + m_0^2\, \sigma_7(t) - m_0^2\, A_t^2\, \sigma_6(t) \\ m_3^2(t) &= \mu_0\, m_{12}\, \sigma_2(t) + B_0\, m_0\, \mu_0\, \sigma_8(t) + \mu_0\, m_0\, A_t\, \sigma_3(t) \end{aligned} \tag{9}$$

The coefficients σ_i depend on $t \equiv \ln M_G^2/Q^2$ with functional dependence given in [7, 25, 26, 27, 28]. The (high scale) boundary values ($t = 0$) are normally chosen to be $\sigma_{1,2,..,6} = 0$, $\sigma_{7,8} = 1$ (i.e. $c_{h_{1,2}} = 1$). For $Q^2 = m_Z^2$ ($t = t_z$) the values of these coefficients are given in Appendix A in terms of the top Yukawa coupling. To simplify notation we will not display the argument t_z in what follows.

The quartic term $\delta |h_2|^4$ is generated radiatively [13, 24]. Including leading log two-loop effects one has

$$\delta = \frac{3\, h_t^4}{g^2\, \pi^2} \left[\ln \frac{M_{\tilde{t}}}{m_t} + \frac{X_t}{4} + \frac{1}{32\pi^2} \Big(3\, h_t^2 - 16\, g_3^2\Big) \Big(X_t + 2 \ln \frac{M_{\tilde{t}}}{m_t}\Big) \ln \frac{M_{\tilde{t}}}{m_t} \right],$$

$$X_t \equiv \frac{2\,(A_t\, m_0 - \mu \cot\beta)^2}{M_{\tilde{t}}^2} \left(1 - \frac{(A_t\, m_0 - \mu \cot\beta)^2}{12\; M_{\tilde{t}}^2}\right). \tag{10}$$

with $M_{\tilde{t}}^2 \equiv m_{\tilde{t}_1}\, m_{\tilde{t}_2}$, and g_3 is the strong coupling.

The minimum conditions for V can be written as:

$$v^2 = -\frac{m^2}{\lambda}, \qquad \left(2\lambda \frac{\partial m^2}{\partial \beta} - m^2\, \frac{\partial \lambda}{\partial \beta}\right)_{\beta=\beta_{\min}} = 0, \tag{11}$$

with the notation $v^2 = v_1^2 + v_2^2$, $\tan\beta = v_2/v_1$, $m_Z^2 = g^2\, v^2/4$ and where:

$$\begin{aligned} m^2 &\equiv \tilde{m}_1^2 \,\cos^2\beta + \tilde{m}_2^2\, \sin^2\beta - m_3^2\, \sin 2\beta \\ \lambda &\equiv \frac{g^2}{8}\,(\cos^2 2\beta + \delta\, \sin^4\beta) + \zeta_1\, \sin 2\beta + \frac{\zeta_2}{4}\, \sin^2 2\beta. \end{aligned} \tag{12}$$

Note that in deriving these expressions we have discarded non-leading log corrections except those to the quartic Higgs coupling where the tree level term is anomalously small.

3.2 Analytical results for fine-tuning

The fine tuning of the EW scale with respect to a set of parameters p introduced in [7] is

$$\Delta \equiv \max \mathrm{Abs}\big[\Delta_p\big]\Big|_{p=\{\mu_0^2, m_0^2, A_t^2, B_0^2, m_{12}^2,\, \zeta_1^2,\, \zeta_2^2\}}, \qquad \Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p} \tag{13}$$

With $m^2 = m^2(p,\beta)$, $\lambda = \lambda(p,\beta)$ we can find $\partial\beta/\partial p$ from the second of eqs.(11) (more precisely this determines the parameter dependence of $\beta_{\min}$):

$$\frac{\partial\beta}{\partial p} = \frac{1}{z}\left(-2\frac{\partial\lambda}{\partial p}\frac{\partial m^2}{\partial\beta} - 2\lambda\frac{\partial^2 m^2}{\partial\beta\partial p} + \frac{\partial m^2}{\partial p}\frac{\partial\lambda}{\partial\beta} + m^2\frac{\partial^2\lambda}{\partial\beta\partial p}\right)$$

where

$$z = \lambda\left(2\frac{\partial^2 m^2}{\partial\beta^2} + v^2\frac{\partial^2\lambda}{\partial\beta^2}\right) - \frac{v^2}{2}\left(\frac{\partial\lambda}{\partial\beta}\right)^2.$$

Using this one finds [14]

$$\Delta_p = -\frac{p}{z}\left[\left(2\frac{\partial^2 m^2}{\partial\beta^2} + v^2\frac{\partial^2\lambda}{\partial\beta^2}\right)\left(\frac{\partial\lambda}{\partial p} + \frac{1}{v^2}\frac{\partial m^2}{\partial p}\right) + \frac{\partial m^2}{\partial\beta}\frac{\partial^2\lambda}{\partial\beta\partial p} - \frac{\partial\lambda}{\partial\beta}\frac{\partial^2 m^2}{\partial\beta\partial p}\right]. \tag{14}$$

3.2.1 A general two-Higgs model

Using eq.(14) we derived a general analytical result for the fine-tuning of the EW scale in a general two-Higgs doublet model allowing for the most general renormalisable Higgs potential, see Appendix B, eq.(B-6). The results are presented in terms of derivatives of the soft masses and couplings of the scalar potential.

3.2.2 The MSSM with dimension-five operators (MSSM_5)

Applied to the case of the MSSM with dimension-five operators the results presented in Appendix B give:

$$\begin{aligned}
\Delta_{\mu_0^2} &= -\frac{1}{v^2\,D}\Big\{v^2\,\cos 2\beta\,\Big[\sin 2\beta\,\Big(\zeta_1\,(\,2\,\gamma_2-\delta\,g^2\,v^2/8)\\
&+\;2\,\gamma_1\,\big[\,\delta g^2/8-\big((4+\delta)\,g^2/8-\zeta_2\big)\;\cos 2\beta\;\big]\Big)\Big]+2\Big[2\mu_0^2\,\sigma_8^2+(\zeta_1\,v^2-\gamma_1)\sin 2\beta\Big]\\
&\times\Big[\gamma_4-v^2\,\big(2\zeta_1\sin 2\beta-\zeta_2\cos 4\beta\big)\Big]\Big\}
\end{aligned}\tag{15}$$

$$\begin{aligned}
\Delta_{m_0^2} &= -\frac{1}{4\,v^2\,D}\Big\{-v^2\zeta_2\sin 4\beta\,\Big[4\gamma_1\;\cos 2\beta+\big(\delta\,g^2\,v^2/8-2\gamma_2\big)\sin 2\beta\Big]\\
&+2\,v^2\Big[2\,(\gamma_1-\mu_0\,m_{12}\sigma_2)\cos 2\beta+\gamma_3\sin 2\beta\Big]\Big[4\zeta_1\cos 2\beta+\delta g^2\cos\beta\sin^3\beta\\
&+\;(\zeta_2-g^2/2)\,\sin 4\beta\Big]+8\,\Big[\gamma_4-v^2\,\big(2\zeta_1\sin 2\beta-\zeta_2\cos 4\beta\big)\Big]\\
&\times\;\Big[2m_0^2-\gamma_3\,\sin^2\beta+\zeta_2\,v^2\sin^2\beta\cos^2\beta-(\gamma_1-m_{12}\,\mu_0\,\sigma_2)\sin 2\beta\Big]\Big\}
\end{aligned}\tag{16}$$

$$\begin{aligned}
\Delta_{m_{12}^2} &= -\frac{m_{12}}{v^2\,D}\,\Big\{\,\frac{v^2}{2}\,\Big[\,2\mu_0\sigma_2\cos 2\beta-\big(A_t\,\sigma_5\;m_0+2\,m_{12}\,(\sigma_4-\sigma_1)\big)\;\sin 2\beta\Big]\\
&\times\;\Big[4\zeta_1\cos 2\beta+\delta\,g^2\cos\beta\sin^3\beta+(\zeta_2-g^2/2)\,\sin 4\beta\Big]+2\,\Big[2\,m_{12}\,\sigma_1-\mu_0\,\sigma_2\sin 2\beta\\
&+\;\big(A_t\,\sigma_5\,m_0+2\,m_{12}(\sigma_4-\sigma_1)\big)\sin^2\beta\Big]\Big[\gamma_4-v^2\,\big(2\zeta_1\sin 2\beta-\zeta_2\cos 4\beta\big)\Big]\Big\}
\end{aligned}\tag{17}$$

$$\begin{aligned}
\Delta_{A_t^2} &= -\frac{A_t}{v^2\,D}\Big\{2\,m_0\sin\beta\,\Big[2\mu_0\,\sigma_3\,\cos\beta+(2\,A_t\,\sigma_6\,m_0-\sigma_5\,m_{12})\sin\beta\Big]\Big[-\zeta_2\,v^2\,\cos 4\beta\\
&+\; 2\,\zeta_1\,v^2\,\sin 2\beta-\gamma_4\Big]+m_0\,v^2\,\Big[\mu_0\sigma_3\cos 2\beta-(1/2)\sigma_5\,m_{12}\,\sin 2\beta+A_t\,\sigma_6\,m_0\sin 2\beta\Big]\\
&\times\; \Big[4\,\zeta_1\cos 2\beta+\delta\,g^2\cos\beta\sin^3\beta+(\zeta_2-g^2/2)\,\sin 4\beta\Big]\Big\}
\end{aligned} \tag{18}$$

$$\begin{aligned}
\Delta_{B_0^2} &= -\frac{2\,B_0\,m_0\,\mu_0\,\sigma_8}{v^2\,D}\Big\{v^2\,\Big[2\zeta_1+(\zeta_2-(4+\delta)g^2/8)\sin^3 2\beta\Big]\\
&\qquad +\,(\delta\,g^2\,v^2/16-\gamma_2)\,\sin 4\beta-4\gamma_1\,\sin^2 2\beta\Big\}
\end{aligned} \tag{19}$$

Also:

$$\begin{aligned}
\Delta_{\zeta_1^2} &= -\frac{\zeta_1}{2\,D'}\Big[2\,m_3^2-2\cos 4\beta\,\,\big(3m_3^2+(4+\delta)\,(g^2v^2/8)\,\sin 2\beta\big)\\
&\qquad +\big(\,3\,(\tilde m_2^2-\tilde m_1^2)+\delta g^2v^2/8\big)\,\sin 4\beta\Big]
\end{aligned} \tag{20}$$

$$\begin{aligned}
\Delta_{\zeta_2^2} &= -\frac{\zeta_2}{8\,D'}\sin 2\beta\,\Big[-2\cos 4\beta\,\big(4\,m_3^2+(4+\delta)\,\,(g^2v^2/8)\,\sin 2\beta\big)\\
&\qquad +\,\big(\,4\,\,(\tilde m_2^2-\tilde m_1^2)+\delta g^2v^2/8\big)\,\sin 4\beta\Big]
\end{aligned} \tag{21}$$

with the notation:

$$D \;\equiv\; 2\Big\{-\frac{1}{8}\,v^2\Big[4\zeta_1\cos 2\beta+\zeta_2\sin 4\beta+g^2\,(\delta\cos\beta\sin^3\beta-1/2\,\sin 4\beta)\Big]^2 \tag{22}$$

$$-\,2\Big[\zeta_1\sin 2\beta+\zeta_2/4\sin^2 2\beta+g^2/8\,(\cos^2 2\beta+\delta\sin^4\beta)\Big]\Big[v^2\,\big(2\zeta_1\sin 2\beta-\zeta_2\cos 4\beta\big)-\gamma_4\Big]\Big\}$$

$$\begin{aligned}
D' &\equiv \frac{g^2}{4}\,\big(\cos^2 2\beta+\delta\sin^4\beta\big)\Big[\big(\,2\,(\tilde m_2^2-\tilde m_1^2)+\delta g^2v^2/8\big)\,\cos 2\beta\\
&-\;\; (4+\delta)\,(g^2v^2/8)\,\cos 4\beta+4\,m_3^2\sin 2\beta\Big]-\frac{g^4v^2}{32}\,\big[2-(4+\delta)\sin^2\beta\big]^2\,\sin^2 2\beta
\end{aligned} \tag{23}$$

and

$$
\begin{aligned}
\gamma_1 &\equiv \mu_0\,(B_0\,m_0\,\sigma_8 + m_{12}\,\sigma_2 + A_t\,m_0\,\sigma_3)\\
\gamma_2 &\equiv (-1+\sigma_7 - A_t^2\,\sigma_6)\,m_0^2 + A_t\,\sigma_5\,m_0\,m_{12} + m_{12}^2\,(\sigma_4-\sigma_1)\,+\delta\,g^2\,v^2/16\\
\gamma_3 &\equiv 2\,(1-\sigma_7 + A_t^2\,\sigma_6)\,m_0^2 - A_t\,\sigma_5\,m_{12}\,m_0\\
\gamma_4 &= 2\,\gamma_2\,\cos 2\beta + 4\gamma_1\,\sin 2\beta - (4+\delta)\,(g^2\,v^2/8)\,\cos 4\beta
\end{aligned}
\tag{24}
$$

The contributions $\Delta_{\zeta_i^2}$ are proportional to ζ_i so, for small enough changes from the MSSM case, the fine-tuning introduced with respect to these new parameters is small and sub-leading relative to that for the other parameters.

It is convenient to treat β as the free parameter rather than B_0. Using the second minimum condition of (11) (after replacing m_3^2 by (9)), one finds

$$
\begin{aligned}
B_0 &= \frac{-1}{m_0\,\mu_0\,\sigma_8}\Big\{\mu_0\,m_{12}\sigma_2\,+\mu_0\,m_0\,A_t\,\sigma_3 - \frac{1}{2}(\tilde m_1^2+\tilde m_2^2)\Big[\sin 2\beta\\
&+\frac{v^2}{\tilde m_1^2+\tilde m_2^2}\Big(\zeta_1\,(1+\sin^2 2\beta)+\frac{\zeta_2}{2}\,\sin 2\beta + \delta\,(g^2/8)\,\sin 2\beta\sin^2\beta\Big)\Big]\Big\}
\end{aligned}
\tag{25}
$$

Note that $\gamma_{1,4}$ brings some extra $\zeta_{1,2}$ dependence through B_0, while $\gamma_{2,3}$ are $\zeta_{1,2}$ independent. Δ_p, $p=\{\mu_0^2, m_0^2, A_t^2, B_0^2, m_{12}^2\}$ contain some $\mathcal{O}(\zeta_i^2)$ terms, although the potential is only linear in ζ_i. In the above expressions all coefficients $\sigma_{1,2,\ldots,8}$ are evaluated at m_Z and their values are given in (A-1). They depend only on the top Yukawa coupling at m_Z. The only approximation in obtaining the above expressions for Δ_p's is that we did not include the effect of derivatives (with respect to parameter p) acting on δ (the radiative correction to the quartic term). This is a legitimate approximation since this effect is numerically very small (for the MSSM alone it induces an error for fine-tuning Δ equal to or less than unity, while in the MSSM_5 the error is even smaller (1%); for larger $\tan\beta$ this error is further reduced).

The above results for the fine-tuning measure simplify in the limit of ignoring the RG effects on the masses i.e. $\sigma_{1,2,3,\ldots,6}=0$; $\sigma_{7,8}=1$.In this case

$$
\begin{aligned}
\gamma_1 &= \mu_0\,B_0\,m_0, \qquad \gamma_2 = \delta\,g^2 v^2/16, \qquad \gamma_3 = 0,\\
\gamma_4 &= 4\,B_0\,m_0\,\mu_0\,\sin 2\beta + (g^2\,v^2/8)\,[\,\delta\cos 2\beta - (4+\delta)\,\cos 4\beta\,]
\end{aligned}
\tag{26}
$$

Ignoring RG effects on the quartic couplings too, $\delta=0$, then $\gamma_4 = 4B_0\,m_0\,\mu_0\sin 2\beta - (g^2\,v^2/2)\,\cos 4\beta$, $\gamma_2=0$. Finally, in the limit $\zeta_{1,2}=0$ of the fine tuning relations Δ_p, one obtains analytical expressions for the EW scale fine tuning in the MSSM alone, with $\gamma_{1,\ldots 4}$ as in (24). Since these may be useful for other studies, they are provided in Appendix A.

3.3 The large $\tan\beta$ limit.

The above formulae for the fine-tuning simplify considerably in the limit of large $\tan\beta$. Ignoring terms suppressed by inverse powers of $\tan\beta$ one has

$$\begin{aligned}
\Delta_{\mu_0^2} &= \frac{-2\mu_0^2\,\sigma_8^2}{(1+\delta)\,m_Z^2} \\
\Delta_{m_0^2} &= \frac{-m_0}{(1+\delta)\,m_Z^2}\left[2\,\sigma_7\,m_0 + A_t\,(\sigma_5\,m_{12} - 2A_t\,\sigma_6\,m_0) + \frac{\mu_0}{m_0}\frac{\zeta_1\,v^2\,m_{12}\,\sigma_2}{\gamma_2 + (1+\delta/4)\,m_Z^2}\right] \\
\Delta^2_{m_{12}} &= \frac{-m_{12}}{(1+\delta)\,m_Z^2}\left[A_t\,\sigma_5\,m_0 + 2\,\sigma_4\,m_{12} - \frac{\zeta_1\,v^2\,\mu_0\,\sigma_2}{\gamma_2 + (1+\delta/4)\,m_Z^2}\right] \\
\Delta_{A_t^2} &= \frac{A_t\,m_0}{(1+\delta)\,m_Z^2}\left[2\,A_t\,\sigma_6\,m_0 - \sigma_5\,m_{12} + \frac{\zeta_1\,v^2\,\mu_0\,\sigma_3}{\gamma_2 + (1+\delta/4)\,m_Z^2}\right] \\
\Delta_{B_0^2} &= \frac{-\zeta_1\,(m_{12}\,\sigma_2 + A_t\,m_0\,\sigma_3)\,\mu_0\,v^2}{(1+\delta)\,m_Z^2\,(\gamma_2 + m_Z^2\,(1+\delta/4))} \\
\Delta_{\zeta_i^2} &= 0, \quad i = 1,2
\end{aligned} \tag{27}$$

One may see that all the fine tuning measures are suppressed by the factor $(1+\delta)^{-1}$ demonstrating why quantum corrections to the quartic Higgs coupling can significantly reduce the fine-tuning. A similar effect applies at small $\tan\beta$ as well.

3.3.1 Dimension-six operators

The operator analysis used here has a limited range of validity because it corresponds to integrating out new heavy degrees of freedom. If the mass of these degrees of freedom is not much above the energies being probed, the operator analysis breaks down and one must deal with the new degrees of freedom directly. The mass, M_*, at which this happens corresponds to the point where high dimension operators are not suppressed relative to low dimension operators. A measure of this may be obtained by dimensional analysis in which the operator matrix elements are taken to be determined by the energy scale being probed. Applied here, this implies that the operator analysis is reliable provided $\frac{m_0,\mu}{M_*} \ll 1$.

A potential fault in this estimate of the range of convergence occurs because higher dimension operators may have anomalously large matrix elements. An example of this occurs for the dimension-six operators listed in Section 2. Consider the first dimension-six operator in eq.(3)

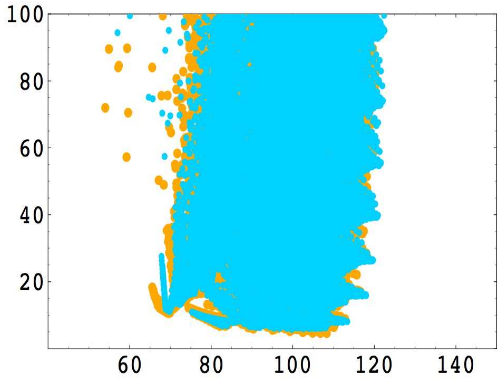
$$\mathcal{O}_2 \supset \frac{c_1}{M_*^2}\Big[S^\dagger S (H_2^\dagger \exp V\, H_2)^2\Big]_D \supset \frac{c_1\, m_0^2}{M_*^2}\, |h_2|^4, \quad (c_1 \sim \mathcal{O}(1)). \tag{28}$$

This should be compared to the leading quartic Higgs term coming from the dimension-five operators in eq.(7) that contributes at $\mathcal{O}\left(\frac{2\mu_0}{M_*}\, |h_2|^2\, h_1 h_2\right)$. One may see that the relative magnitude of the dimension-six to dimension-five contributions is $\mathcal{O}\left(\frac{m_0^2}{2\mu_0 M_*}\tan\beta\right)$. Thus, strictly, the region of validity of the dimension-five operator analysis is $\frac{m_0^2}{2\mu_0 M_*}\tan\beta \ll 1$. However, as discussed in the next section, the new physics generating this dimension-six operator is different from that generating the dimension-five operator and so their coefficients should be uncorrelated. In this case the addition of higher dimension operators can reduce the fine tuning for some region in parameter space so that the analysis with dimension-five operators only will provide a useful upper bound even in regions where dimension-six contributions are significant. For this reason the region of validity of the dimension-five operators analysis is better described by the original $m_0/M_* \ll 1$ and $\mu_0/M_* \ll 1$ condition. This keeps the corrections coming from operators with correlated coefficients small. To see this more explicitly consider the effect of the term in eq.(28) on fine-tuning. Writing $\nu \equiv c_1\, m_0^2/M_*^2$ one has

$$\Delta_p \approx -\frac{2\,p}{(1+\delta+8\nu/g^2)\, m_Z^2}\frac{\partial \tilde{m}_2^2}{\partial p} \tag{29}$$

where, for example, p can be μ_0^2. The partial derivative is readily obtained from eq.(9). The dominant effect of the d=6 operator on fine tuning is the appearance of the effect of the $8\nu/g^2$ term in the denominator, reducing the fine tuning for the appropriate sign of $8\nu/g^2$. (note that c.f. eq.(27) such a term is not generated by the dimension-five term at large $\tan\beta$). The effect of this reduction is sizeable. Taking, for example, $m_0/M_* \approx 1/10$ and $c_1 = 3$, then $8\nu/g^2 \approx 1/2$ which is close to the numerical value of δ entering the denominator. One sees that $d = 6$ operators can bring a reduction of $\Delta_{\mu_0^2}$ relative to that of the MSSM including top/stop effects, of order $(8\nu/g^2)/(1+\delta+8\nu/g^2) \approx 30\%$.

In the following numerical analysis we include only the effects of dimension-five operators. The convergence criterion found above gave $m_0/M_* \ll 1$ and $\mu_0/M_* \ll 1$. In our following numerical analysis this bound is comfortably satisfied when we take m_0/M_*, $\mu_0/M_* \leq 0.035$, giving upper values $\zeta_{1,2} \leq 0.07$.



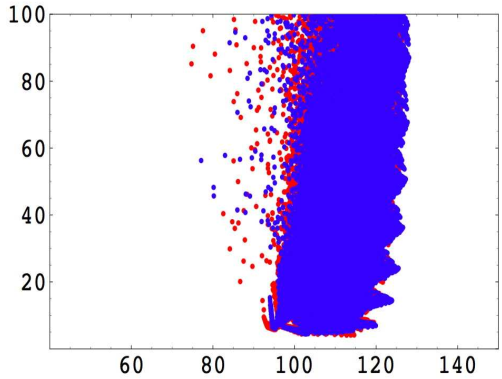


Figure 1: Left figure (a): the MSSM fine tuning Δ as a function of m_h; Right figure (b): the fine tuning in the MSSM with $d=5$ operators in terms of m_h, with $\zeta_1=\zeta_2=0.03$. In both figures, the top pole mass considered is $m_t = 174$ GeV for blue (dark blue) areas and $m_t = 171.2$ for yellow (red) areas, respectively. Larger m_t input (blue) shifts the plots towards higher m_h by 2-5 GeV. In both figures the parameters space scanned is: $1.5 \leq \tan\beta \leq 10$, $50\,\text{GeV} \leq m_0, m_{12} \leq 1$ TeV, $130\,\text{GeV} \leq \mu_0 \leq 1$ TeV, $-10 \leq A_t \leq 10$.

3.4 Numerical results

We are now in a position to determine the fine-tuning in the extended MSSM Higgs sector. We will present this as a function of the mass m_h of the lightest CP even Higgs. This is given by:

$$m_h^2 = \frac{1}{2}\Big[m_A^2 + m_Z^2 - \sqrt{w} + \xi\Big] + \zeta_1\, v^2 \sin 2\beta \left[1 + \frac{m_A^2 + m_Z^2}{\sqrt{w}}\right] + \frac{\zeta_2\, v^2}{2}\left[1 - \frac{(m_A^2 - m_Z^2)\cos^2 2\beta}{\sqrt{w}}\right] \tag{30}$$

where

$$w \equiv [(m_A^2 - m_Z^2)\cos 2\beta + \xi]^2 + \sin^2 2\beta\,(m_A^2 + m_Z^2)^2$$
$$m_A^2 = \tilde{m}_1^2 + \tilde{m}_2^2 + \xi/2 + \zeta_1\, v^2 \sin 2\beta - (1/2)\,\zeta_2\, v^2; \quad \xi \equiv \delta m_Z^2 \sin^2\beta \tag{31}$$

Using the results of the previous section we compute the fine tuning for a sample of points in parameter space in the region with: $1.5 \leq \tan\beta \leq 10$, $50\,\text{GeV} \leq m_0, m_{12} \leq 1$ TeV, 130 GeV $\leq \mu_0 \leq 1$ TeV, $-10 \leq A_t \leq 10$ and $171.2 \leq m_t \leq 174$ GeV consistent with $m_t = 172.6 \pm 1.4$ GeV [29], and with the signs for $\zeta_{1,2}$ chosen so as to reduce the fine tuning.

The results are shown in Figures 1 to 3. Note that in these figures the structure apparent at small Δ and large m_h is probably a scanning artefact. We expect the under-dense wedge

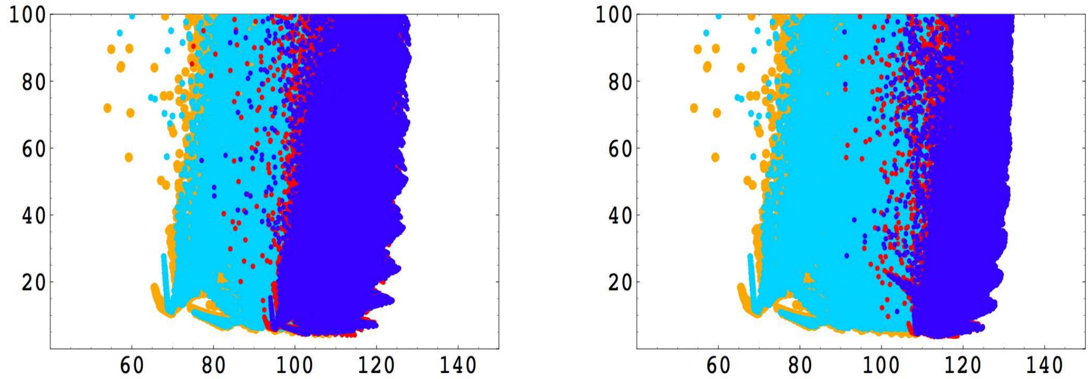


Figure 2: Left figure (a): the fine tuning Δ as a function of m_h. Δ of MSSM is plotted in light blue (m_t = 174 GeV) with an orange edge (m_t = 171.2); Δ of MSSM with d=5 operators with $\zeta_1 = \zeta_2 = 0.03$ is plotted in dark blue (m_t = 174 GeV) with a red edge (m_t = 171.2). Right figure (b): similar to figure (a) but with $\zeta_1 = \zeta_2 = 0.05$. Non-zero or larger ζ_i (dark blue and red areas) shift the plots to higher m_h to allow a reduced Δ for higher m_h. In both figures $1.5 \le \tan\beta \le 10$; $50\,\text{GeV} \le m_0, m_{12} \le 1$ TeV, $130\,\text{GeV} \le \mu_0 \le 1$ TeV, $-10 \le A_t \le 10$.

shaped regions will be filled in with a more dense parameter sample. Similarly at very large Δ, corresponding to very precise relationships between parameters, there will be some points corresponding to high values of m_h that are not picked up by our finite parameter scan.

Turning to our results, as a benchmark Figure 1(a) shows the EW scale fine tuning Δ of eq.(13) of the MSSM as a function of m_h. One may see that $\Delta \ge 18$ for values $m_h \ge 114.4$ GeV, the current LEPII bound. Figure 1(b) shows Δ for the case of the MSSM with dimension-five operators added, with $\zeta_1 = \zeta_2 = 0.03$. The dominant effects in Figure 1(b) are mostly due to the effect of non-zero ζ_1, which comes from the supersymmetric part of the higher dimensional operator. One may see a systematic shift of the allowed region to higher m_h which (for positive ζ_i) is driven by an increase in the quartic Higgs coupling which appears in the denominator of the fine tuning measure ($c.f.$ eq.(27)). The overall result is that the minimum amount of fine-tuning Δ in the presence of $d = 5$ effective operators is small, of order $\Delta \approx 6$, for m_h from 95 to 119 GeV. Therefore non-zero ζ_i can accommodate larger m_h while keeping a Δ significantly smaller than in the MSSM. To illustrate the change more directly we superpose both plots in Figure 2 (a). The effect is enhanced for larger operator coefficients, as may be seen in Figure 2 (b), where Δ is presented for $\zeta_1 = \zeta_2 = 0.05$, again shown relative to the MSSM case. One can see that in this case values of $m_h > 114.4$ GeV can have a low $\Delta \approx 6$. Therefore Δ can be significantly reduced from the MSSM case, for a

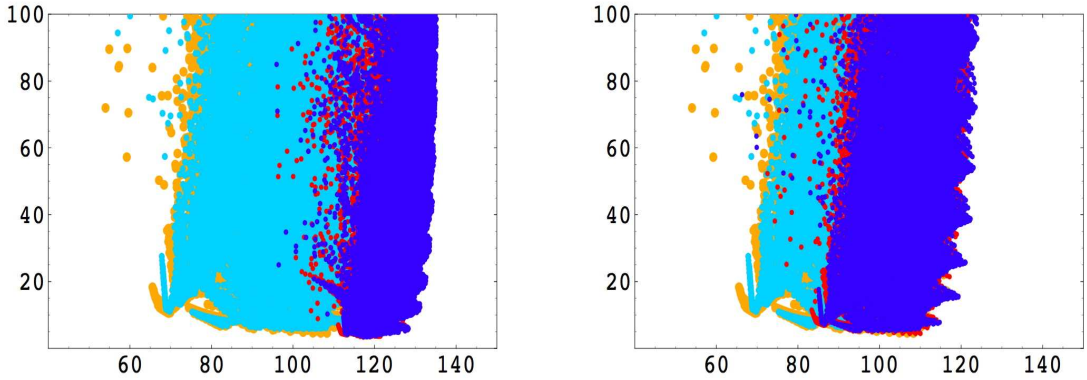


Figure 3: As in Fig.2 but with: left figure (a): $\zeta_1 = 0.07$, $\zeta_2 = 0$; right figure (b): $\zeta_1 = 0$, $\zeta_2 = 0.1$

similar m_h. This conclusion is further supported by the plots in Figure 3 where other values for ζ_i are considered. From all plots shown one sees that $\Delta < 10$ is easily satisfied for values of the Higgs mass that can be as large as 130 GeV, depending on the exact values of ζ_i.

Note that, in the MSSM, Δ increases for low $\tan\beta$ $(\ll 10)$ and m_h above the LEPII bound. However, c.f. eq.(30), the effect of the $d = 5$ operators is important for low $\tan\beta$ and in their presence Δ actually decreases for low $\tan\beta$. Thus the reduction in the fine tuning at very low $\tan\beta$ relative to the MSSM case is much more marked than that shown.

The lower amount of fine tuning in the presence of effective operators is due to two effects. The first, already mentioned, is the presence of additional quartic Higgs couplings enhancing the denominator which determines the Higgs via $v^2 = -m^2/\lambda$ thus allowing for a smaller electroweak breaking scale. The second is the fact that higher dimensional operators add a tree level contribution to the Higgs mass, which reduces the need for large quantum contributions, and therefore the fine tuning.

What is the scale of new physics needed for this reduction in fine tuning? Using eq.(8) we find that the scale of new physics is

$$M_* \approx 2\mu_0/\zeta_1 \approx (40 \ \text{to} \ 65) \times \mu_0, \qquad \zeta_{1,2} = 0.05 \ \text{to} \ 0.03 \tag{32}$$

With μ_0 between the EW scale and 1 TeV, this shows that large values of M_* are allowed: $M_* \approx (5.2 \text{ to } 8.45)$ TeV for $\mu_0 = 130$ GeV and $M_* \approx (8 \text{ to } 13)$ TeV for $\mu_0 = 200$ GeV. For larger μ_0 one obtains values of M_* above the LHC reach. Finally, for $\zeta_1 = 0.07$ but with $\zeta_2 = 0$, one has $M_* \approx 30 \times \mu_0$ and $\Delta < 10$ for $m_h \approx 130$ GeV. Thus, the EW fine tuning is small $\Delta < 10$ for $114 \leq m_h \leq 130$ GeV, for rather conservative values of $\zeta_{1,2}$. To relax these values one can use that an increase of ζ_1 by 0.01 increases m_h by 2 to 4 GeV for the same Δ.

3.5 The origin of "new physics"

The presence of a higher dimension operator signals new physics and it is important to ask what this new physics can be. In the context of new renormalisable interactions it may come from the effects of new chiral superfields or from new gauge vector superfields. Consider chiral superfields first. One may readily obtain the $d = 5$ operator of eq.(1) by integrating out a gauge singlet or a triplet [20]. Consider the case of a massive gauge singlet X with Lagrangian

$$\mathcal{L}_X = \int d^4\theta \, X^\dagger X + \left\{ \int d^2\theta \left[\mu H_1 H_2 + \lambda_x \, X \, H_1 H_2 + \frac{1}{2} \, M_* \, X^2 \right] + h.c. \right\}.$$

For $M_* \gg \mu$, m_0, one may use the eqs of motion to integrate out X, giving, to leading order in inverse powers of M_*,

$$\mathcal{L}_X^{effective} = \frac{-\lambda_x^2}{2\, M_*} \int d^2\theta \, (H_1 H_2)^2 + h.c. \tag{33}$$

The supersymmetry breaking terms associated with this operator are obtained by replacing $\lambda \rightarrow \lambda(S)$ giving the $d = 5$ operator of interest. Note that $\mathcal{L}_X$ has a similar form to that of the NMSSM. However in the NMSSM the singlet field has mass of order the electroweak breaking scale and cannot be integrated out whereas here we are taking the singlet mass to be much larger than the EW scale.

However, the origin of the $d = 5$ operator cannot be uniquely ascribed to a gauge singlet field. Indeed it may equally well point to the existence of $SU(2)$ triplets [30, 30, 31, 32] $T_{1,2,3}$ of hypercharge $\pm 1, 0$. In this case a Lagrangian of the form

$$\mathcal{L}_T = \int d^4\theta \left[T_1^\dagger e^V T_1 + T_2^\dagger e^V T_2 \right] + \int d^2\theta \left[\mu H_1 H_2 + M_* T_1 T_2 + \lambda_1 H_1 T_1 H_1 + \lambda_2 H_2 T_2 H_2 \right] + h.c$$

gives, to lowest order in $1/M_*$, eq.(33) with λ_x^2 replaced by $\lambda_1 \lambda_2$. More generally, one can generate the $d = 5$ operator through a combination of both gauge singlets and triplets. However note that the pure singlet X case has the advantage of not affecting the gauge couplings unification (at one-loop), which is not true for the $SU(2)$ triplet.

What about additional, massive, $SU(2)$ doublets that couple to the MSSM Higgs sector? One may readily show that integrating them out does not generate, to lowest order in $1/M_*$, an operator of the type (33).

There remains the possibility that the new physics is due to the effect of new massive vector gauge superfields. The simplest example is the case there is a new $U(1)'$ gauge symmetry under which the Higgs sector is charged. This brings extra quartic contributions to the scalar potential that are expected to reduce the fine-tuning [12, 33, 34]. Assuming the $U(1)'$ is broken at M_* one obtains the effective Lagrangian to leading order in inverse powers of M_* given by

$$\mathcal{L}_{U(1)}^{effective} = -\frac{g^{'2}}{M_*^2} \int d^4\theta \left[q_1 H_1^\dagger e^V H_1 + q_2 H_2^\dagger e^V H_2 \right]^2$$

where g' is the $U(1)'$ coupling and $q_{1,2}$ are the charges of the Higgses under $U(1)'$ ($q_1 + q_2 = 0$). Note that, after including the associated supersymmetry breaking operators, this corresponds to the $d = 6$ effective operators [20] of eq.(3) and that no $d = 5$ operators are generated.

In summary, the requirement that the SUSY extension of the MSSM should not have significant fine tuning may indicate the presence of the $d = 5$ operator of eq.(1) which, in turn, suggests the presence of a massive gauge singlet and/or a $SU(2)$ triplet. This is the simplest interpretation based on new renormalisable interactions but other, more complicated possibilities to generate the $d = 5$ operator may be possible.

3.6 Further remarks on fine tuning

Effective field theory approaches to the fine tuning of the electroweak scale were used before in models of low susy breaking scale scenarios [14] where both $d = 5$ and $d = 6$ operators were included. The model in [14] introduces supersymmetry breaking through coupling of MSSM states to a SM singlet field responsible for supersymmetry breaking. After integrating this field out, in addition to the $d = 5$ operator considered here, there are correlated contributions from the $d = 6$ operators. Using this, the authors find the fine tuning can be very small even for an arbitrarily high Higgs mass, provided the scale of supersymmetry breaking is less than 500 GeV.

How does this analysis relate to the one presented here? The examples given in [14] are found varying the ratio $\tilde{m}/M$ in the range 0.05 to 0.8 where $\tilde{m}$ is the supersymmetry breaking scale and M is the messenger mass. For $\tilde{m}/M$ small, the fine tuning is close to that in the MSSM but reduces rapidly for $\tilde{m}/M$ large; in this latter case the fine tuning actually reduces as the Higgs mass increases. This range of values for $\tilde{m}/M$ corresponds to a choice of our

m_0/M_* and μ_0/M_* in a similar range. The upper value strongly violates our criterion for applicability of the operator analysis and is a factor of ≈ 10 larger than the value chosen in Figure 3(a). Ignoring, for the moment, the fact that the contributions of higher dimension operators are expected to be large for this choice of mediator mass, we can ask what this choice of mediator mass in our analysis would give for the Higgs mass consistent with small Δ. Since the change in our upper bound on the Higgs mass roughly scales with the coefficient of the $d = 5$ operator (eq.(30)), this would allow a Higgs mass in the region of 276 GeV, much larger than our earlier conservative estimates. However, as we have stressed, for this value of the messenger mass the operator analysis breaks down and one should do the analysis including the messenger fields explicitly.

4 Conclusions

The LEPII lower bound on the Higgs mass places MSSM Higgs physics at the forefront of supersymmetry phenomenology. While this bound can be satisfied by including the MSSM quantum corrections, it (re)introduces some amount of fine tuning in the model. To reduce the fine tuning may require new physics beyond the MSSM which can be parametrised by higher dimensional operators. In this paper we used an effective field theory framework with $d = 5, 6$ operators in the MSSM Higgs sector, and presented a model independent approach to the fine tuning problem.

We obtained exact analytical results for the EW scale fine tuning in the MSSM with dimension-five operators, which are also applicable to the pure MSSM case in the limit the coefficients of the higher dimension operators vanish. This calculation included one-loop corrections to the soft masses and dominant top Yukawa effects on the quartic terms of the potential. Similar analytical results were given for a general two-Higgs doublet model.

Fine tuning proves to be very sensitive to the addition of higher dimensional operators and this is mostly due to extra corrections to the quartic couplings of the Higgs field. For the case of dimension-five operators we showed that one can maintain a reduced fine-tuning $\Delta < 10$ for a Higgs mass above the LEPII bound and as large as $m_h \approx 130$ GeV, for the parameter space considered, with low $\tan\beta$ ($\tan\beta < 10$). The scale of new physics M_* responsible for the reduction in fine tuning can be rather large, for example $M_* \approx 2\mu_0/\zeta_1 \approx (40\,\text{to}\,65) \times \mu_0$, for $\zeta_{1,2} = 0.05$ to 0.03, and $M_* \approx 30 \times \mu_0$ for $\zeta_1 = 0.07$, $\zeta_2 = 0$. For values of μ_0 between the

electroweak scale and 1 TeV, these results show that large values of M_* are allowed; in the former case $M_* \approx$ (5.2 to 8.45) TeV for $\mu_0 = 130$ GeV and $M_* \approx$ (8 to 13) TeV for $\mu_0 = 200$ GeV. For larger μ_0, larger values of M_* are possible, even above the LHC reach. These results follow from rather conservative choices for the coefficients of the quartic couplings induced by the dimension-five operators, to ensure the convergence of the effective operator expansion.

Our numerical analysis included the effect of dimension-five operators only. These give the leading corrections at low $\tan\beta$, being proportional to $1/M_*$. However, dimension-six operators, suppressed by m_0^2/M_*^2 or μ_0^2/M_*^2, give contributions that can be enhanced by large $\tan\beta$; for $(\tan\beta\, m_0)/M_* > 1$ or $(\tan\beta\, \mu_0)/M_* > 1$ these will be the leading terms. For this reason they should be included at large $\tan\beta$ and we hope to extend our analysis to the dimension-six case in the future.

Of course the crucial question is what is the origin of the physics beyond the MSSM giving rise to these operators? The dimension-five operator can be generated by a gauge singlet superfield or a $SU(2)$ triplet superfield of mass of $\mathcal{O}(M_*)$ coupling to the Higgs sector. The dimension-six operators can be generated, for example, by an extra gauge symmetry with a massive gauge supermultiplet or additional (Higgs-like) $SU(2)$ doublet supermultiplets of mass $\mathcal{O}(M_*)$. If the fine tuning criterion is indeed of physical relevance, the significant amount of fine tuning found in the MSSM already indicates the need for such additional degrees of freedom.

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Appendix

A Fine tuning expressions in the MSSM.

The coefficients σ_i used in the text, Section 3.1 are:

$$
\begin{aligned}
\sigma_1(t_z) &= 0.532, & \sigma_2(t_z) &= 0.282\,(4.127\,h_t^2 - 2.783)(1.310 - h_t^2)^{1/4} \\
\sigma_3(t_z) &= -0.501\,h_t^2\,(1.310 - h_t^2)^{1/4}, & \sigma_4(t_z) &= 0.532 - 5.233\,h_t^2 + 1.569\,h_t^4 \\
\sigma_5(t_z) &= 0.125\,h_t^2\,(10.852\,h_t^2 - 14.221), & \sigma_6(t_z) &= -0.027\,h_t^2\,(10.852\,h_t^2 - 14.221) \\
\sigma_7(t_z) &= 1 - 1.145\,h_t^2, & \sigma_8(t_z) &= 1.314\,(1.310 - h_t^2)^{1/4}
\end{aligned}
\tag{A-1}
$$

where h_t is evaluated at m_Z and $m_t = h_t(t_{m_t})\,(v/\sqrt{2})\sin\beta$.

In the MSSM one obtains the following analytical expressions for fine-tuning (these are obtained from the results in Section 3.1 by setting $\zeta_{1,2} = 0$):

$$
\Delta_{\mu_0^2} = -\frac{1}{v^2\,D}\Big\{(g^2v^2/8)\,\gamma_1\,\sin 4\beta\big[\,\delta - (4+\delta)\,\cos 2\beta\,\big] + 2\,\big[\,2\mu_0^2\,\sigma_8^2 - \gamma_1\sin 2\beta\,\big]\gamma_4\Big\} \tag{A-2}
$$

$$
\begin{aligned}
\Delta_{m_0^2} &= -\frac{1}{4\,v^2\,D}\Big\{2\,v^2\Big[2\,(\gamma_1 - \mu_0\,m_{12}\sigma_2)\cos 2\beta + \gamma_3\sin 2\beta\Big]\Big[\delta g^2\cos\beta\sin^3\beta - (g^2/2)\,\sin 4\beta\Big] \\
&+ 8\,\gamma_4\Big[2m_0^2 - \gamma_3\,\sin^2\beta + (m_{12}\,\mu_0\,\sigma_2 - \gamma_1)\,\sin 2\beta\Big]\Big\}
\end{aligned}
\tag{A-3}
$$

$$
\begin{aligned}
\Delta_{m_{12}^2} &= -\frac{m_{12}}{v^2\,D}\,\Big\{\frac{g^2\,v^2}{2}\,\Big[2\mu_0\sigma_2\cos 2\beta - \big(A_t\,\sigma_5\,m_0 + 2\,m_{12}\,(\sigma_4 - \sigma_1)\big)\,\sin 2\beta\Big]\Big[\delta\,\cos\beta\sin^3\beta \\
&- \frac{1}{2}\sin 4\beta\Big] + 2\,\Big[2m_{12}\,\sigma_1 - \mu_0\sigma_2\sin 2\beta + \big(A_t\,\sigma_5\,m_0 + 2\,m_{12}(\sigma_4 - \sigma_1)\big)\sin^2\beta\Big]\gamma_4\Big\}
\end{aligned}
\tag{A-4}
$$

$$
\begin{aligned}
\Delta_{A_t^2} &= -\frac{A_t}{v^2\,D}\,\Big\{2\,m_0\sin\beta\,\big[\,2\mu_0\,\sigma_3\,\cos\beta + (2\,A_t\,\sigma_6\,m_0 - \sigma_5\,m_{12})\sin\beta\,\big]\,(-\gamma_4) \qquad \text{(A-5)} \\
&+ \big[\,\delta\cos\beta\sin^3\beta - \frac{1}{2}\sin 4\beta\,\big]\,\big[\,\mu_0\,\sigma_3\cos 2\beta - \sigma_5\,m_{12}/2\,\sin 2\beta + A_t\,\sigma_6\,m_0\sin 2\beta\big]m_0\,g^2v^2\Big\}
\end{aligned}
$$

and

$$\Delta_{B_0^2} = -\frac{2\,B_0\,m_0\,\mu_0\,\sigma_8}{v^2\,D}\left\{(\delta g^2\,v^2/16-\gamma_2)\,\sin 4\beta-(4+\delta)\frac{g^2 v^2}{8}\sin^3 2\beta-4\gamma_1\,\sin^2 2\beta\right\} \quad \text{(A-6)}$$

The denominator D is now

$$D \;=\; \frac{1}{4}\,g^2\left\{-\,g^2\,v^2\,(\delta\cos\beta\sin^3\beta-1/2\,\sin 4\beta)^2-2\,(\cos^2 2\beta+\delta\sin^4\beta)(-\gamma_4)\right\} \quad \text{(A-7)}$$

with the notation

$$\begin{aligned}
\gamma_1 &\equiv \mu_0\,(B_0\,m_0\,\sigma_8+m_{12}\,\sigma_2+A_t\,m_0\,\sigma_3)\\
\gamma_2 &\equiv (-1+\sigma_7-A_t^2\,\sigma_6)\,m_0^2+A_t\,\sigma_5\,m_0\,m_{12}+m_{12}^2\,(\sigma_4-\sigma_1)\,+\delta\,g^2\,v^2/16\\
\gamma_3 &\equiv 2\,(1-\sigma_7+A_t^2\,\sigma_6)\,m_0^2-A_t\,\sigma_5\,m_{12}\,m_0\\
\gamma_4 &\equiv 4\gamma_1\sin 2\beta+2\gamma_2\cos 2\beta-(4+\delta)\,(g^2\,v^2/8)\cos 4\beta
\end{aligned} \quad \text{(A-8)}$$

and finally

$$B_0 = \frac{1}{m_0\,\mu_0\,\sigma_8}\left\{\frac{1}{2}\,(\tilde m_1^2+\tilde m_2^2)\,\sin 2\beta\left[1+\frac{\delta\,g^2\,v^2/8}{\tilde m_1^2+\tilde m_2^2}\sin^2\beta\right]-\mu_0\,m_{12}\sigma_2\,-\mu_0\,m_0\,A_t\,\sigma_3\right\} \quad \text{(A-9)}$$

The results for fine tuning given above considered a common bare gaugino mass, but this restriction can easily be lifted to obtain similar expressions.

B Evaluation of fine-tuning Δ_p in general two-Higgs doublet models.

We present here the analytical result for the EW fine-tuning wrt a parameter p, for an arbitrary two-Higgs doublet model. This can be immediately applied to a specific model. Start with the general potential

$$\begin{aligned}
V &= \tilde m_1^2\,|H_1\,|^2+\tilde m_2^2\,|H_2\,|^2-(m_3^2\,H_1\cdot H_2+h.c.)\\
&+\frac{1}{2}\,\lambda_1\,|H_1\,|^4+\frac{1}{2}\,\lambda_2\,|H_2\,|^4+\lambda_3\,|H_1\,|^2\;|H_2\,|^2\;+\lambda_4\,|\,H_1\cdot H_2\,|^2\\
&+\left[\frac{1}{2}\,\lambda_5\,(H_1\cdot H_2)^2+\lambda_6\,|\,H_1\,|^2\,(H_1\cdot H_2)+\lambda_7\,|\,H_2\,|^2\,(H_1\cdot H_2)+h.c.\right]
\end{aligned} \quad \text{(B-1)}$$

In the particular case of MSSM with $d=5$ operators

$$\lambda_1 = \frac{1}{4}(g_2^2+g_1^2), \qquad \lambda_2 = \frac{1}{4}(g_2^2+g_1^2)(1+\delta), \qquad \lambda_3 = \frac{1}{4}(g_2^2-g_1^2)$$
$$\lambda_4 = -\frac{1}{2}g_2^2, \qquad \lambda_5 = \zeta_2, \qquad \lambda_6 = \lambda_7 = \zeta_1 \tag{B-2}$$

while in the MSSM alone one also sets $\zeta_1 = \zeta_2 = 0$.

The minimum conditions can be written

$$-v^2 = \frac{m^2}{\lambda}, \qquad 2\lambda \frac{\partial m^2}{\partial \beta} - m^2 \frac{\partial \lambda}{\partial \beta} = 0 \tag{B-3}$$

with

$$\begin{aligned} m^2 &= \tilde{m}_1^2 c_\beta^2 + \tilde{m}_2^2 s_\beta^2 - m_3^2 s_{2\beta} = \tilde{m}_2^2 - \frac{2}{u}m_3^2 + \frac{1}{u^2}(\tilde{m}_1^2 - \tilde{m}_2^2) + \mathcal{O}(1/u^3) \\ \lambda &= \frac{\lambda_1}{2}c_\beta^4 + \frac{\lambda_2}{2}s_\beta^4 + (\lambda_3+\lambda_4+\lambda_5)s_\beta^2 c_\beta^2 + 2\lambda_6 c_\beta^3 s_\beta + 2\lambda_7 c_\beta s_\beta^3 \\ &= \frac{\lambda_2}{2} + \frac{2}{u}\lambda_7 + \frac{1}{u^2}(\lambda_3+\lambda_4+\lambda_5-\lambda_2) + \mathcal{O}(1/u^3), \end{aligned} \tag{B-4}$$

with $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, $u \equiv \tan\beta = v_2/v_1$, $h_i = 1/\sqrt{2}\,(v_i + \tilde{h}_i)$, $m_Z^2 = (g_1^2+g_2^2)\,v^2/4$. At large $\tan\beta$:

$$-v^2 = \frac{2\tilde{m}_2^2}{\lambda_2} + \frac{1}{u\,\lambda_2^2}(-4\,m_3^2\lambda_2 - 8\,\lambda_7\,\tilde{m}_2^2) + \mathcal{O}(1/u^2); \tag{B-5}$$

Definition (14) obtained using (B-3) can be used to find the most general result Δ_p for the EW fine-tuning wrt a parameter p. This takes account of the dependence $\beta = \beta(p)$ induced by the min conditions. One finds the general expression:

$$\Delta_p = \frac{\partial \ln v^2}{\partial \ln p} = \frac{-\left\{2\,w_1'\,z_1 - (1/4)\,z_1'\,w_2 + [w_3' + (1/v^2)\,z_2']\,[z_3 + v^2\,w_4]\right\}}{-(1/32)\,v^2\,w_2^2 - w_3\,[-z_3 - w_4\,v^2]} \tag{B-6}$$

with the following notations:

$$
\begin{aligned}
w'_1 &\equiv \lambda'_6 \cos^4\beta + \lambda'_{3451}\,\cos^3\beta\sin\beta - \frac{3}{4}\,(\lambda'_6-\lambda'_7)\,\sin^2 2\beta - \lambda'_{3452}\,\cos\beta\sin^3\beta - \lambda'_7\,\sin^4\beta,\\
w_2 &\equiv 4\,(\lambda_6+\lambda_7)\cos 2\beta + 4\,(\lambda_6-\lambda_7)\,\cos 4\beta - 2[\lambda_1-\lambda_2+(\lambda_1+\lambda_2-2\,\lambda_{345})\cos 2\beta]\,\sin 2\beta\\
w_3 &\equiv \frac{1}{2}\,\lambda_1\cos^4\beta + 2\lambda_6\,\cos^3\beta\sin\beta + \frac{1}{4}\lambda_{345}\sin^2 2\beta + 2\lambda_7\cos\beta\sin^3\beta + \frac{1}{2}\lambda_2\sin^4\beta\\
w'_3 &\equiv \frac{1}{2}\,\lambda'_1\cos^4\beta + 2\lambda'_6\,\cos^3\beta\sin\beta + \frac{1}{4}\lambda'_{345}\sin^2 2\beta + 2\lambda'_7\cos\beta\sin^3\beta + \frac{1}{2}\lambda'_2\sin^4\beta\\
w_4 &\equiv -(\lambda_1+\lambda_2-2\lambda_{345})\,\cos 4\beta - 2\,(\lambda_6+\lambda_7)\,\sin 2\beta + 4\,(\lambda_7-\lambda_6)\,\sin 4\beta\\
z_1 &\equiv -2m_3^2\cos 2\beta + (\tilde m_2^2 - \tilde m_1^2)\,\sin 2\beta,\\
z'_1 &\equiv -2(m_3^2)'\cos 2\beta + [(\tilde m_2^2)' - (\tilde m_1^2)']\,\sin 2\beta\\
z'_2 &\equiv (\tilde m_1^2)'\cos^2\beta + (\tilde m_2^2)'\sin^2\beta - (m_3^2)'\,\sin 2\beta,\\
z_3 &\equiv \left[4\,\tilde m_2^2 - 4\,\tilde m_1^2 + (\lambda_2-\lambda_1)\,v^2\right]\,\cos 2\beta + 8\,m_3^2\,\sin 2\beta
\end{aligned}
\qquad \text{(B-7)}
$$

where $\lambda_{345} = \lambda_3+\lambda_4+\lambda_5$; $\lambda_{345j} \equiv \lambda_3+\lambda_4+\lambda_5-\lambda_j$, $(j=1,2)$ and with:

$$
(m_3^2)' \equiv \frac{\partial m_3^2}{\partial \ln p}, \quad (\tilde m_j^2)' \equiv \frac{\partial \tilde m_j^2}{\partial \ln p}, \ \ (j=1,2); \quad \lambda'_i \equiv \frac{\partial \lambda_i}{\partial \ln p}, \ \ i=1,2...7. \qquad \text{(B-8)}
$$

The general result (B-6), (B-7) can be applied to any two-Higgs doublet model, which includes all radiative corrections in the couplings and soft masses. The result in (B-6) simplifies considerably in most cases, since usually many λ_i are independent of p, i.e. have $\lambda'_i = 0$.

It is worth taking some particular limits of the above result for Δ_p. At large $\tan\beta$:

$$
\begin{aligned}
\Delta_p &= \frac{1}{v^2}\left\{\frac{v^2\,[\,4\,\lambda_7\,(m_3^2)' - 4\lambda'_7\,m_3^2 + \lambda'_2\,(\,2\,\tilde m_1^2 - 2\,\tilde m_2^2 + \lambda_{3452}\,v^2\,)]}{-2\,\lambda_2\,(\tilde m_1^2-\tilde m_2^2) + [-\lambda_2\lambda_{3452} + 2\lambda_7^2\,]\,v^2}\right.\\
&\qquad \left. + \frac{2\,[\,2\,(\tilde m_1^2-\tilde m_2^2) + \lambda_{3452}\,v^2]\,(\tilde m_2^2)'}{-2\,\lambda_2\,(\tilde m_1^2-\tilde m_2^2) + [-\lambda_2\lambda_{3452} + 2\lambda_7^2\,]\,v^2}\right\} + \mathcal{O}(1/\tan\beta)
\end{aligned}
\qquad \text{(B-9)}
$$

which for $\lambda'_2 = \lambda'_7 = 0$ gives

$$
\Delta_p = \frac{-2}{\lambda_2\,v^2}\left\{(\tilde m_2^2)' - \frac{2\lambda_7\,v^2\,\left[\lambda_2\,(m_3^2)' + \lambda_7\,(\tilde m_2^2)'\right]}{2\lambda_7^2\,v^2 - \lambda_2\,[\lambda_{3452}\,v^2 + 2\,(\tilde m_1^2-\tilde m_2^2)]}\right\} + \mathcal{O}(1/\tan\beta) \qquad \text{(B-10)}
$$

In MSSM $\lambda_7 = \lambda'_7 = 0$ then

$$
\Delta_p = \frac{-2}{\lambda_2\,v^2}\,(\tilde m_2^2)' + \mathcal{O}(1/\tan\beta) \qquad \text{(B-11)}
$$

which is consistent with (B-5) and the definition of Δ_p (assuming $\delta' = 0$).
In MSSM with $d = 5$ operators, $\lambda_7 = \zeta_1$ so

$$\Delta_p = -\frac{2}{v^2\,(1+\delta)\,m_Z^2}\left[(\tilde m_2^2)' - \frac{\zeta_1\,v^2}{\tilde m_2^2 - \tilde m_1^2 + m_Z^2\,(1+\delta/2)}(m_3^2)'\right] + \mathcal{O}(1/\tan\beta) \qquad \text{(B-12)}$$

which recovers the results of (27).

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